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A TWO-EDDY THEORY OF PREMIXED TURBULENT FLAME PROPAGATION

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CONTENTS

Available experimental data on the turbulent burning velocity of premixed gases are surveyed. There is discussion of the accuracy of experimental measurements and the means of ascertaining relevant turbulent parameters. Results are presented in the form of the variation of the ratio of turbulent to laminar burning velocities with the ratio of r.m.s. turbulent velocity to laminar burning velocity, for different ranges of turbulent Reynolds number. A two-eddy theory of burning is developed and the theoretical predictions of this approach, as well as those of others, are compared with experimentally measured values.

Nomenclature

A	constant, equation (22)
b	bar width
B	dimensionless volumetric rate of burning
c	constant of proportionality for eddy chemical lifetime
L	integral scale of turbulence
n	eddy number density
p	probable fraction of an eddy that burns during the eddy lifetime

Re pipe flow Reynolds number
R_L $u'L/\nu$
r mean reactedness
r_f final reactedness
t time
t_f time to attain r_f
t_η Kolmogorov time scale
u_l laminar burning velocity
u_t turbulent burning velocity
u' r.m.s. turbulent velocity
U mean flow velocity
V volume of an eddy
x distance downstream from mid-plane of screen
γ intermittency factor
η Kolmogorov microscale
ϵ turbulent diffusivity
δ characteristic length scale, equation (30)
δ_l laminar flame thickness
δ_t turbulent flame thickness
κ eddy decay constant
λ Taylor microscale of turbulence
ν kinematic viscosity
τ dimensionless time (u'^2t/ν)
τ_c dimensionless chemical lifetime
τ_d dimensionless eddy lifetime
τ_f dimensionless time to attain r_f

Subscripts

L large eddy
p parallel-sided channel flow
n small eddy

1. Introduction

On the basis of a variety of experimental evidence the authors of this paper have proposed that, for the combustion of turbulent premixed gases, the ratio of turbulent to laminar burning velocity, u_t/u_l, can be correlated with the ratio of the r.m.s. turbulent velocity of reactant gas to its laminar burning velocity, u'/u_l, and with the turbulent Reynolds number R_L based upon the integral length scale, L (Abdel-Gayed & Bradley 1977*a*). Some theoretical support also has been demonstrated for this correlation (Abdel-Gayed *et al.* 1979), which extends over a wide range of gaseous fuels, equivalence ratios and types of combustor.

Partly because of the difficulties, which are now being overcome through laser-Doppler velocimetry, of measuring turbulence throughout the flame gases, it has been desirable to express parameters of turbulence in terms of those for the cold premixture. Clearly, a satisfactory theory of turbulent burning should be able to correlate all the burning velocity data which have been obtained over a wide range of pertinent parameters in a variety of burners. This was first attempted previously (Abdel-Gayed & Bradley 1977*a*), but before discussing the correlations presented in this paper it is convenient to outline the sources of error in the various measured parameters. These are:

(i) Most values of *laminar burning velocity* have been measured experimentally and most of the various techniques have associated errors. These have been discussed previously in some detail (Andrews & Bradley 1972*a*). For some conditions accurate values are not available.

(ii) The errors in the measurement of *turbulent burning velocity* are probably greater.

(iii) Particularly in burners, *departures from isotropy and uniformity* occur. It follows that many of the data are for conditions different from the postulated ideal. Usually the turbulent parameters were measured under cold flow conditions, and the effect of combustion upon turbulence in the cold premixture was unknown.

(iv) Accurate measurements of *r.m.s. turbulent velocity* can be made but the measurements of *turbulent length scales*, where they have been made, are probably not as accurate. In several instances they have not been measured at all, and in some few instances neither has the r.m.s. turbulent velocity.

In § 2 all available experimental data from burners, from a variety of sources, are presented in the terms of the three recommended dimensionless groups. In several cases it was necessary to estimate parameters that had not been measured in the original investigations, and the limitations of the available data are discussed. Some of the data have not been presented in this form before, and where it is necessary to estimate turbulent parameters, the relations used to do this are more accurate than those used previously. Turbulent burning during explosions manifests some differences from steady-state combustion and will be the subject of a later paper.

In § 3 a theoretical approach to small-eddy burning is developed. This is applicable at the higher levels of turbulence and is an extension of the authors' two-eddy theory of burning (Abdel-Gayed *et al.* 1979). A particular problem in the analysis arises from the limited understanding of the intermittent high frequency structure in a turbulent flame. In § 4 the predictions of this and other theoretical approaches are compared with the experimental data of § 2.

2. Experimental measurements of turbulent burning velocity

The available experimental data for a wide range of conditions are summarized in table 1. All the turbulent data apply to the cold gas.

Reference to the table shows that experimentalists have recognized the importance of measuring the r.m.s. turbulent velocity but that often the turbulent length scale has not been measured. The importance of kinematic viscosity has also been unrecognized. Few workers have presented data that are complete enough for the present dimensionless correlation. An alternative method of deriving the turbulent Reynolds number and u', therefore, is developed for circular tube and flat slot burners. Experimental measurements of turbulent length scales and r.m.s. turbulent velocities in non-reacting flows in pipes and between parallel plates are correlated with the flow Reynolds numbers, Re and Re_p for pipes and plates, respectively.

One of the problems with such burners is that u' and L vary across the burner. Townes *et al.* (1972) and Powe & Townes (1973) have measured the variations of these parameters across a pipe, and Laufer (1951) has measured them between parallel plates. In the present work the values of these are estimated, for the direction normal to the flow, at the plane half-way between the central axis and the walls, because flame measurements have been associated with this region. Such estimates are not facilitated by the increasing departure from isotropic turbulence and the lack of length-scale data as the walls are approached.

Experimental measurements of r.m.s. turbulent velocity, by hot wire anemometry in non-reacting flows across a pipe diameter, have been measured by other workers (Townes *et al.* 1972; Laufer 1954; Mickelsen 1955; Baldwin & Walsh 1961; Lawn 1971; Robertson *et al.* 1965). Figure 1 shows the variation with *Re* of the ratio of the r.m.s. turbulent velocity for the radial direction to the mean centre-line axial velocity, u'/U, at the half radius of the pipe. The relation is expressed by

$$u'/U = 0.168(Re)^{-0.119} \quad \text{for} \quad 3\times 10^4 < Re < 7\times 10^5. \tag{1}$$

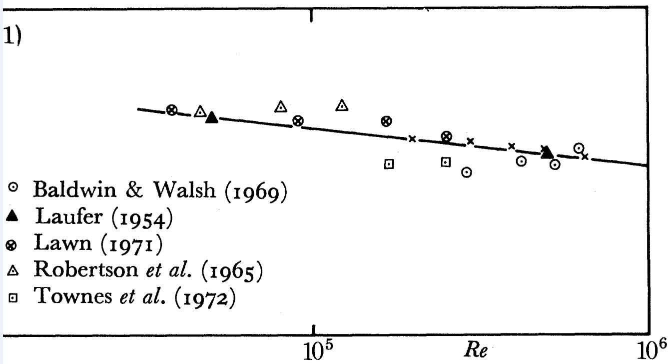

FIGURE 1. Variation of radial r.m.s. turbulent velocity at half radius of the pipe, with flow Reynolds number.

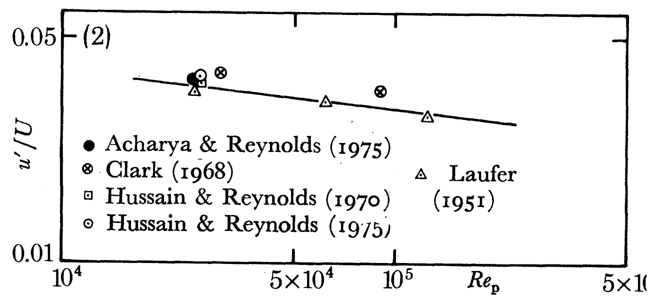

FIGURE 2. Variation of radial r.m.s. turbulent velocity at midpoint between the centre plane and plate, with channel flow Reynolds number.

Similarly, figure 2 shows the variation of this ratio half-way between the centre plane and a plate with values of Re_p that are based upon the distance between the parallel plates. This relation is expressed by

$$u'/U = 0.115(Re_p)^{-0.118} \quad \text{for} \quad 2\times 10^4 < Re_p < 2\times 10^5. \tag{2}$$

The lack of data for L remote from the centre-line of a pipe makes it impossible to obtain directly the variation of R_L with *Re*. In this case, the isotropic relation between the turbulent transport number, ϵ/ν, and R_L, recently proposed by the authors of this paper (Abdel-Gayed & Bradley 1980), is adopted. This is given by

$$\epsilon/\nu = 0.49 R_L^{0.97} \quad \text{for} \quad 70 < R_L < 3000. \tag{3}$$

The variation of ϵ/ν for momentum transport across a pipe is given by the expression of Reichardt (1951). At half the radius of the pipe this yields

$$\epsilon/\nu = 0.0075(Re)^{\frac{7}{8}}. \tag{4}$$

From equations (3) and (4),

$$R_L = 13.45\times 10^{-3}(Re)^{0.902} \quad \text{for} \quad 70 < R_L < 3000. \tag{5}$$

However, in the present work this equation is used in the range $16 < R_L < 3520$.

For parallel plates, the measurements made by Laufer (1951) for L and u' are used to obtain R_L, and these values are shown in figure 3. To extend the range of Reynolds number, equation (3) has been applied to the measurements of ϵ/ν made by Page *et al.* (1952*a*, *b*), Venezian & Sage

(1961) and Hussain & Reynolds (1975) to obtain the corresponding values of R_L. These are also shown in figure 3. The relation of the full line is

$$R_L = 15.30 \times 10^{-3}(Re_p)^{0.911} \quad \text{for} \quad 10 < R_L < 1000. \tag{6}$$

This equation is used in the range $72 < R_L < 4600$.

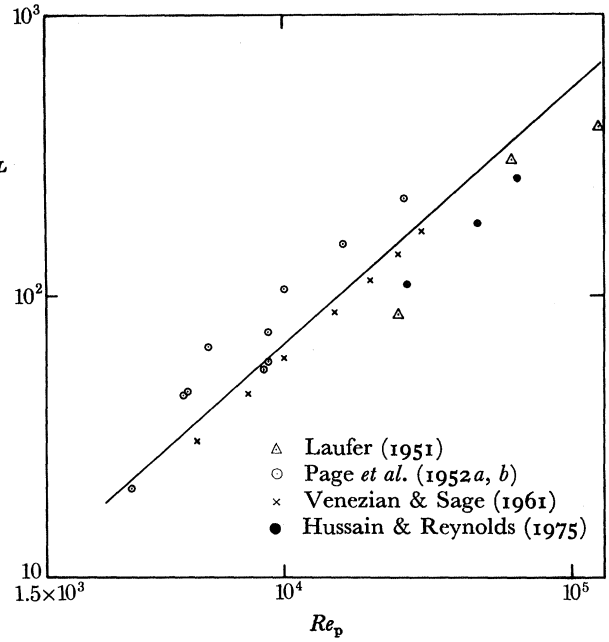

FIGURE 3. Variation of radial turbulent Reynolds number at midpoint between the centre plane and plate, with channel flow Reynolds number.

Where it is necessary to estimate the value of L in grid-generated turbulence, the data of Baines & Peterson (1951) are used in the form

$$L/b = 0.155(x/b)^{0.452}, \tag{7}$$

where L again applies to the direction transverse to the flow.

The experimental data, summarized in table 1, have been processed to give figures 4–16, in decreasing order of Reynolds number. These show values of u_t/u_l plotted against u'/u_l for thirteen different restricted ranges of R_L. There is considerable scatter of experimental points, which possibly largely reflects the sources of error discussed in § 1.

Examination of the data in these figures suggests there was no significant effect due to flame confinement when the flame was confined in ducts of widths down to twice the burner diameter. Even with a duct width equal to the burner diameter, as in the work of Kozachenko & Kuznetsov (1965), the values of u_t/u_l tend to be only slightly higher. This increase might be explained by enhanced turbulence generation through shear stresses. In table 1 the term 'confined flame' refers to one which is within a duct with a width less than twice the burner diameter.

The least-square fit to a second-order polynomial has been drawn on each graph. In deriving these curves the results of Petrov & Talantov (1959) have been discarded in the mathematical optimization. These investigators worked with benzene–air mixtures at 373 K, for which they also measured u_l. However, the authors of this paper opine that these values are too low. The

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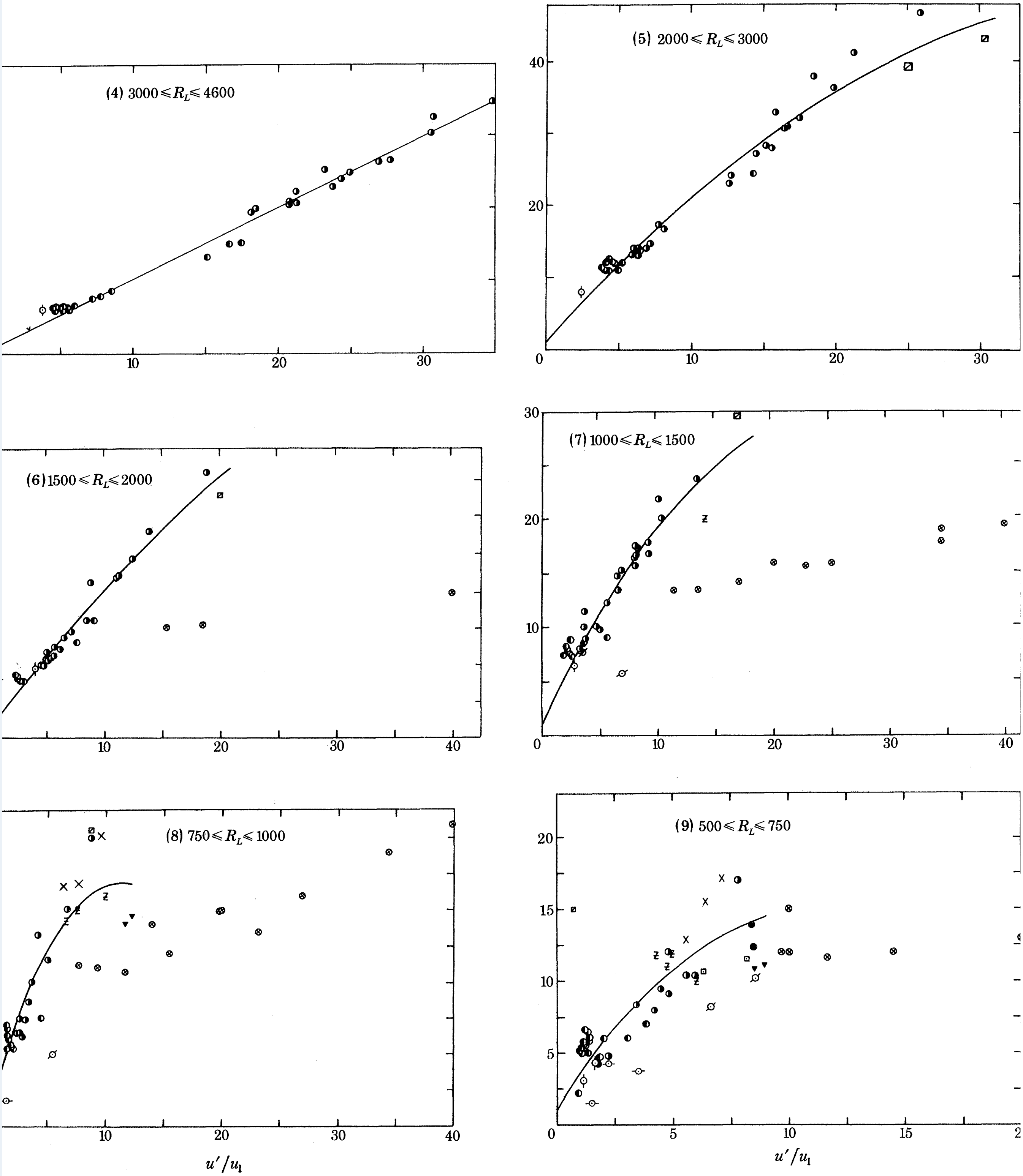
TABLE 1. SOURCES OF TURBULENT BURNING VELOCITY DATA

authors	method	mixture	P/atm†	T/K	u_l/(m s^{-1})	u_l reference	u'/(m s^{-1})	L/mm	$10^6\nu$/(m^2 s^{-1})
Ballal (1979)	duct, grid, 'V' flame, angle, stabilizing pilot flame	C_3H_8–air C_2H_2–air H_2–air	0.2	290	0.2–2.8	Ballal (1979)	measured, 0.3–9.8	measured 0.15–51.1	150
Ballal & Lefebvre (1975)	square tube, grid, 'V' flame, angle, stabilizing pilot flame	C_3H_8–air	1	298	0.45	Ballal & Lefebvre (1975)	measured, 0.25–3.84	measured, 0.56–5.08	14.72
Bollinger & Williams (1949)	tube, total area, stabilizing pilot flame at high flow rate	C_2H_2–air C_2H_4–air C_3H_8–air	1	298	0.45–1.47	Bollinger & Williams (1949)	equation (1), 0.162–2.85	equation (5), R_L	14.72–15.68
Damköhler (1940)	tube, inner area	C_3H_8–O_2	1	298	1.27–2.90	Damköhler (1940)	equation (1), 1.12–4.17	equation (5), R_L	11.6–13.2
Grover *et al.* (1963)	tube, grid, total area	CH_4–air	1	298	0.23	Grover *et al.* (1963)	given, 0.19–0.65	given, 0.76–1.27	15.89
Karlovitz (1954)	tube, inner angle	CH_4–air	1	298	0.45	Andrews & Bradley (1972*b*)	measured, 0.09–3.5	measured, 0.5–3	15.88
Karlovitz *et al.* (1951)	tube, inner angle, $r/R \leqslant 0.7$	CH_4–air C_2H_2–air	1	298	0.45, 1.75	Andrews & Bradley (1972*b*) Karlovitz *et al.* (1951)	measured, 0.4–0.65	equation (5), R_L	15.88
Khramtsov (1959)	tube, grid, inner area, stabilizing pilot flame	C_3H_8–air	0.1–0.6	293	$0.46P^{-0.5}$ 0.59–1.45	Andrews & Bradley (1973) Lindow (1968)	measured, $7\% \leqslant u'/U \leqslant 12.5\%$	measured, 2.8	26.22–157.32
Kozachenko (1960*a*, *b*)	duct, grid, inner area, stabilizing pilot flame, confined	C_6H_6–air	1	440–470	0.69–0.79	Kozachenko (1960*a*) Gibbs & Calcote (1959)	measured, $5\% \leqslant u'/U \leqslant 15\%$	equation (7), 1.54–3.30	29.62–33.1
Kozachenko (1962)	duct, inner area, stabilizing pilot flame, confined	C_6H_6–air	1	290–500	0.46–0.96	Kozachenko (1962) Gibbs & Calcote (1959)	measured, $u'/U = 5\%$	equation (5), R_L	15.27–36.5
Kozachenko & Kuznetsov (1965)	slot burner, inner area, stabilizing pilot flame, confined	C_3H_8–air	1	298	0.27–0.46	Gibbs & Calcote (1959)	measured, 0.8–0.33	equations (2), (6), R_L	14.37–14.85
Kozachenko & Kuznetsov (1965)	slot burner, inner area, stabilizing pilot flame, confined	H_2–air	1	298	0.66–2.42	Andrews & Bradley (1973)	measured, 0.4–11.6	equations (2), (6), R_L	17.83–20.75
Kuznetsov & Malanov (1964)	square tube, 'V' flame, gas velocity angle, stabilizing pilot flame	C_3H_8–air	1	298	0.28–0.46	Gibbs & Calcote (1959)	measured, 0.7–3.22	equation (5), R_L	14.72
Petrov & Talantov (1959)	pipe, grid, inner area, stabilizing pilot flame	C_6H_6–air	1	373	0.2–0.6	Petrov & Talantov (1959)	measured, 1.3–16	equation (7), 2.67–3.90	23.2

† atm ≈ 10^5 Pa.

Fuhs (1962)	flame area, stabilizing pilot flame					Fuhs (1962)	0.02–0.04	3.71	
Rasbash & Rogowski (1960)	tube, pressure and ionization gauges, curve extrapolated to give $U/U_0 = 2.5$ at $Re = 2500$, $u_t/u_l = (2.5)^{-1}(U/U_0)$	C_3H_8–air C_5H_{12}–air	1	298	0.442	Rasbash & Rogowski (1960)	equation (1), 0.201–1.83	equation (5), R_L	*ca.* 15.0
Richmond *et al.* (1957)	tube 'V' flame, angle, stabilizing pilot flame	CH_4–air	1	298	0.45	Andrews & Bradley (1972*b*)	measured, 0.04–0.37	equation (5), R_L	15.89
Singh (1975)	square tube, pressure distribution along the axis of the combustion chamber, using indirect method, hot gas stabilization, confined	CH_4–air	1	559–583	$10+3.71 \times 10^{-4}\,T^2$	Andrews & Bradley (1972*b*)	measured, 7.53–15.37	measured, 3	47.23–50.54
Smith & Gouldin (1978)	tube, grid, 'V' flame, turbulent flame speed measurements were conducted using hot-film and laser anemometry and thermocouple measurements	CH_4–air	1	298	0.25–0.45	Andrews *et al.* (1975*b*)	measured, 0.094–1.12	measured, 0.69–1.49	14.426–15.892
Vinckier & Van Tiggelen (1968)	opposed flow, flame position	$CH_4+2O_2+4.5N_2$ $CH_4+2O_2+2.79N_2$ $CH_4+2O_2+1.96N_2$ $2C_2H_2+5O_2+27.18N$ $2C_2H_2+5O_2+18.48N_2$ $2C_2H_2+5O_2+14.88N_2$	1	298	0.89–1.98	Vinckier & Van Tiggelen (1968)	measured, 1.06–5.70	measured, 0.55–1.64	15.01–16.07
Wagner (1955)	tube, mean flame-surface area, stabilizing pilot flame	C_3H_8–air C_3H_8, O_2, Ar C_3H_8, O_2, He	1	298	0.36–1.24	Wagner (1955)	measured, $2\% \leqslant u'/U \leqslant 4.9\%$	equation (5), R_L	*ca.* 15.0
Williams *et al.* (1949)	nozzle, grid, inner flame area, flame holder, confined	city gas–air	1	298	0.24–0.35	Williams *et al.* (1949)	measured, $u'/U = 2.3\%$	measured, 3.18	16.0
Wohl & Shore (1955)	tube, total area of maximum luminosity, stabilizing pilot flame	CH_4–air C_5H_{12}–air	1	298	0.12–0.40	Wohl & Shore (1955)	given, $0.9\% \leqslant u'/U \leqslant 13.3\%$	given, 0.22–1.64	15.7
Zotin & Talantov (1966*a*)	tube, inner area, stabilizing pilot flame	gasoline–air	1	423–823	0.8–2.92	Inozemtsev (1958)	equation (1), 1.47–4.86	equation (5), R_L	28.1–87.5
Zotin & Talantov (1966*b*)	tube, inner area, stabilizing pilot flame	gasoline–air	1	423–623	0.5–1.65	Inozemtsev (1958)	measured, 1–8	equation (5), R_L	28.1–55.8



FIGURES 4–9. Experimental data for u_t/u_l, u'/u_l. For meaning of symbols, see key below figure 16.

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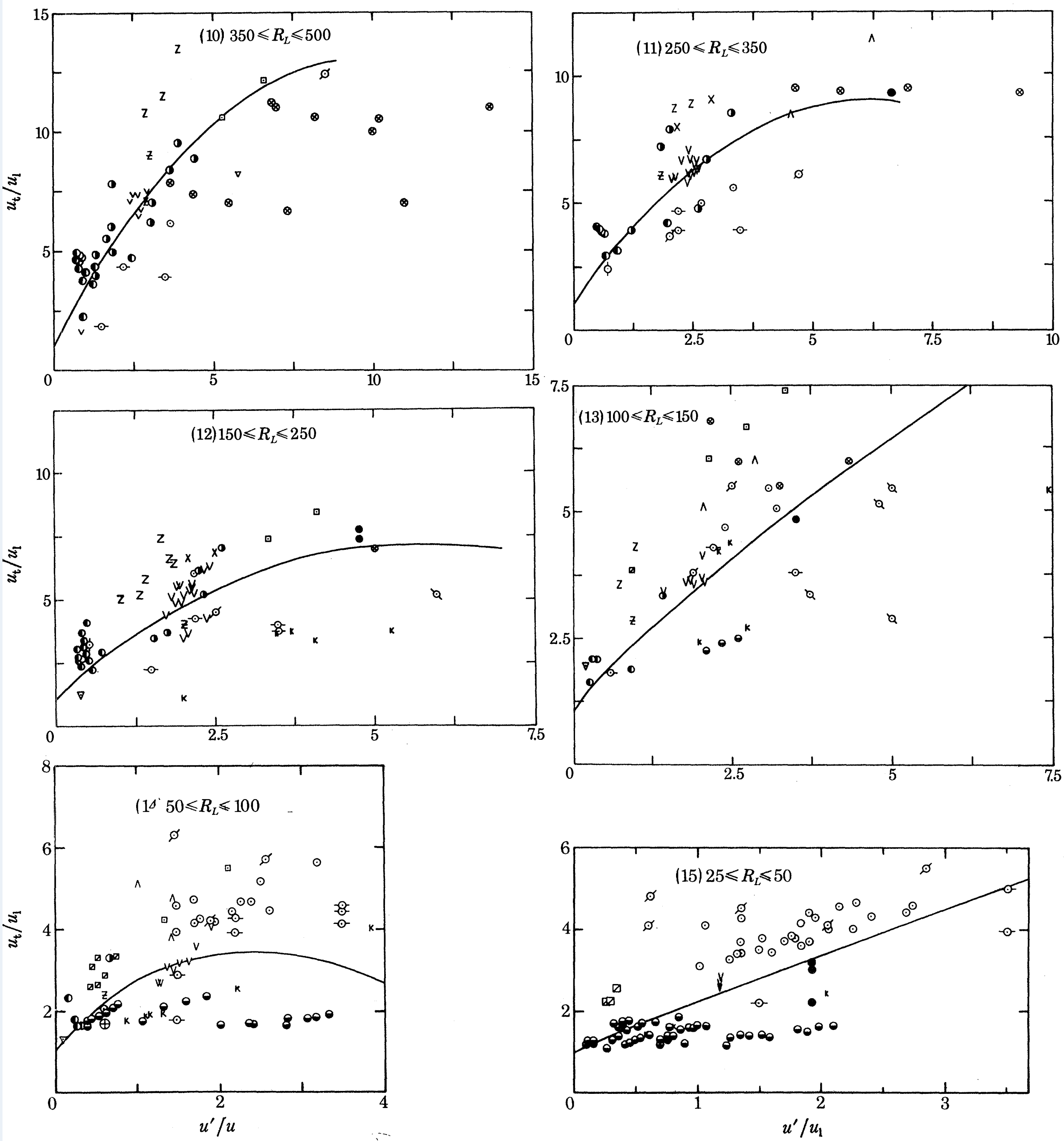

FIGURES 10–15. Experimental data for u_t/u_l, u'/u_l. For meaning of symbols, see key below figure 16.

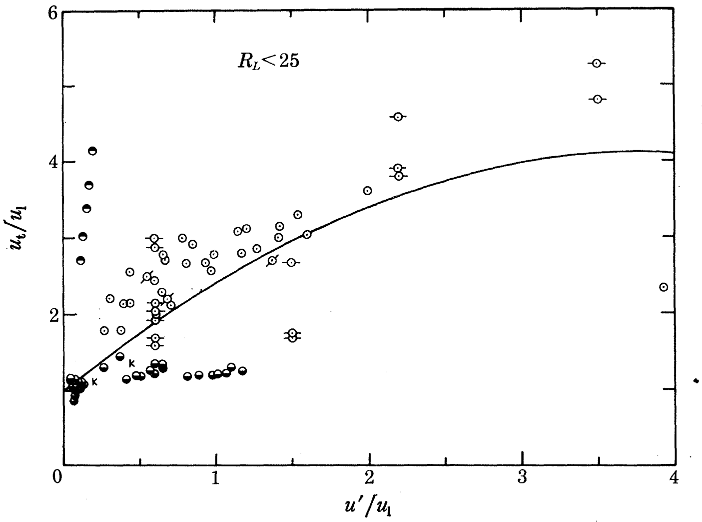

FIGURE 16. Experimental data for u_t/u_l, u'/u_l. For meaning of symbols, see key below.

KEY TO FIGURES 4–16

symbol	reference	symbol	reference
-○-	Ballal (1979)	◓	Povinelli & Fuhs (1962)
ø	Ballal & Lefebvre (1975)	⏀	Rasbash & Rogowski (1960)
κ	Karlovitz (1954)	∇	Richmond *et al.* (1957)
κ	Karlovitz *et al.* (1951)	⊙	Smith & Gouldin (1978)
●	Khramtsov (1959)	▼	Singh (1975)
◘	Kozachenko (1960*a, b*)	v	Vinckier & Van Tiggelen (1968)
◪	Kozachenko (1962)	◒	Wagner (1955)
◑	Kozachenko & Kuznetsov (1965) (propane)	⍉	Wohl & Shore (1955)
◐	Kozachenko & Kuznetsov (1965) (hydrogen)	∧	Williams *et al.* (1949)
x	Kuznetsov & Malanov (1964)	z	Zotin & Talantov (1966*a*)
⊗	Petrov & Talantov (1959)	ƶ	Zotin & Talantov (1966*b*)

curves for the different ranges of R_L are presented in figure 17. The bars on this figure show the standard deviation for these experimental data points. The curves suggest an increasing influence of turbulent length scale, embodied in the turbulent Reynolds number, as the value of u'/u_l is increased. The length scale is often not known with sufficient accuracy and the investigations of Smith & Gouldin (1978) on the effects of this parameter are particularly useful in confirming this trend.

Also shown in figure 17, by the broken curves, are two correlations for ranges of R_L of 2000–3000 and 350–500. which have been presented previously (Abdel-Gayed & Bradley 1977*a*).

Some experimental points for $R_L < 100$ have not been plotted. This is because it is not wished to extrapolate equation (1) into a régime within which it might not be valid. Where the r.m.s. turbulent velocity has been measured, the experimental points are shown. There is considerable scatter in the results for low Reynolds numbers. It is possible that phenomena in this régime are more complex, as a consequence of the onset of turbulence and the presence of instabilities.

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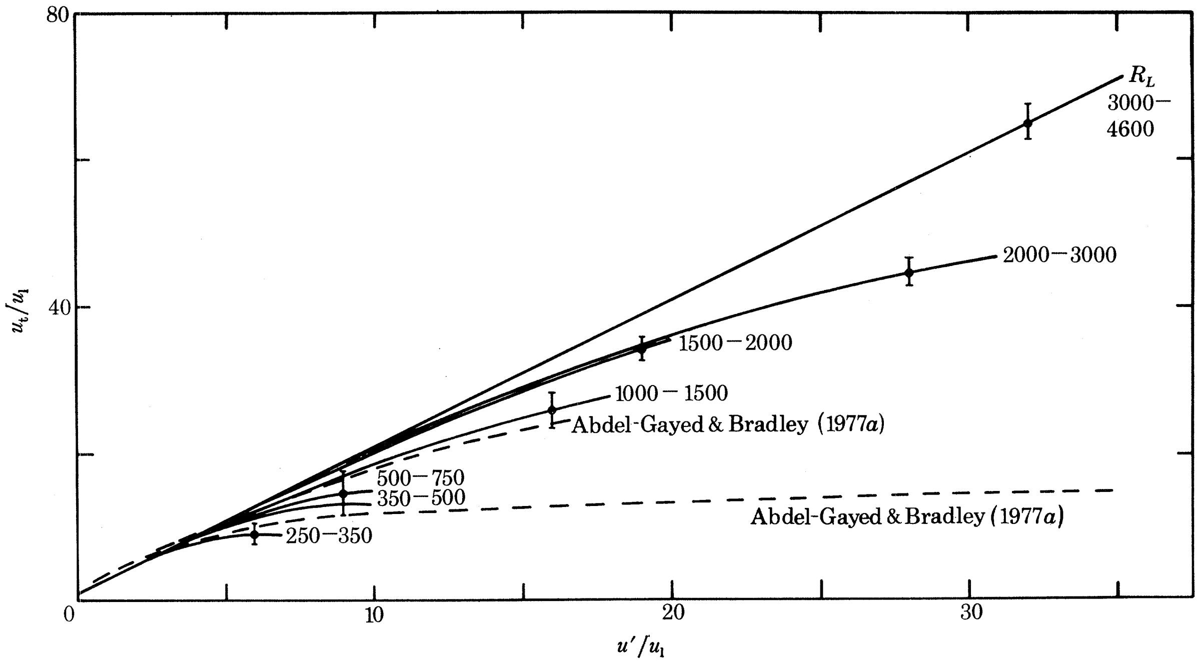

FIGURE 17. Variation of experimental values of u_t/u_l with u'/u_l for different values of R_L.

3. Two-eddy theory of burning

(a) *Theories of turbulent burning*

Many of the experimental data have been obtained at rather low levels of turbulence. This has led to the development of wrinkled flame theories, the different contributions to which have been summarized by Andrews *et al.* (1975*a*). Such theories have commonly required the integral length scale to be greater than the laminar flame thickness, a condition which, indeed, is fulfilled in almost all turbulent flames. An additional condition, sometimes specified, has been that the laminar burning velocity should be greater than the r.m.s. turbulent velocity.

The increase in burning velocity due to turbulence might be attributed to the greater surface area of the outer boundary of the flame. However, as turbulence develops, the small-scale turbulence begins to influence the mechanisms of flame propagation and further enhances the burning rate (Povinelli & Fuhs 1962). Kuznetsov (1976) and Williams (1976) have suggested a criterion for the existence of a wrinkled laminar flame structure as being that the Kolmogorov microscale, η, must be greater than the laminar flame thickness, δ_l.

Kuznetsov (1976) has presented a theoretical analysis of the premixed turbulent flame, based upon the concept of surface combustion. Here, the surface includes not only the outer boundary, but also internal surfaces within this. He pointed out that, because combustion occurs in a reaction zone of width much less than the turbulent length-scale, the governing laws for diffusion of an active species concentrated in that zone are different from those for a more uniformly dispersed inactive species. Equations were formulated in terms of coefficients of molecular diffusion, thermal diffusivity and kinematic viscosity, all of which were assumed to be constant and equal. The use of turbulent transport coefficients was avoided by a Fourier transformation of the unknown probability density function (p.d.f.) for reactedness and velocity. Solution of the equations yielded an analytical expression for turbulent burning velocity. Although it had been assumed that $\eta > \delta_l$, Kuznetsov's expression is similar to that obtained by Zimont & Sabel'nikov (1975) from qualitative considerations and for $\eta < \delta_l$.

Clavin & Williams (1979) have developed a statistical theory applicable to wrinkled flames. Their analysis involves a perturbation for small values of the ratio δ_l/L.

Libby *et al.* (1979) have presented values of turbulent burning velocity based upon perturbation analysis of the rigorously formulated Favre averaged conservation equations. Turbulent transport was described by a conventional eddy viscosity expression and a Damköhler number that related a turbulent to a chemical time. A single-step chemical reaction was assumed, with a rate specified by a global expression. A reaction progress variable was defined and closure of the chemical source term was achieved by specification of a p.d.f. Further extensions to the theory have been presented by Libby & Bray (1980).

Other approaches, since the work of Damköhler (1940), have formulated the problem with less mathematical rigour, but with an emphasis on physical models drawn from both experiment and intuition. Currently this approach still appears to be necessary to some degree, to effect closure in the more mathematical formulations.

In their spectral theory of flame propagation, Povinelli & Fuhs (1962) superposed two terms, one based upon the frequency distribution of large-scale wrinkling and the other upon that of the small-scale influences. This led to an increase in burning velocity by virtue, in the former case, of an increase in flame area and, in the latter case, of an enhanced small-scale turbulent transport.

Ballal & Lefebvre (1975) identified three distinct régimes of turbulent combustion. For high intensities of turbulence, the continuous wrinkled flame surface was replaced by a fragmented reaction zone comprising a fairly thick matrix of burnt gases interspersed with eddies of turbulent mixture. Expressions were presented for burning velocities in the different régimes.

In the eddy break-up theory of Spalding (1971), the rate of decay of large eddies was seen as a rate-determining step, and a similar approach is also featured in the eddy dissipation model of Magnussen & Hjertager (1977). The present two-eddy theory of burning expresses rates in terms of both eddy decay and chemical reaction for two principal groups of eddies within the turbulent spectrum: the large and the dissipative eddies.

(*b*) *Assumptions of the two-eddy theory*

With increasing turbulence, vortex stretching of large eddies produces smaller scales of turbulence. These are associated with increased vorticity and velocity gradients until, through the agency of viscous action, the energy is finally dissipated in randomized molecular motion. In uniform, isotropic turbulence, which is assumed in the theory, the large and small eddy structures are in a state of dynamic equilibrium.

Experimental evidence supports the view that as the level of turbulence increases, so the combustion increasingly is concentrated within smaller length scales (Abdel-Gayed *et al.* 1979). In the two-eddy theory the spectrum of turbulence is simplified into the two principal eddy sizes: large eddies of a size given by the integral length scale, with a lifetime of u'/L, and small dissipative eddies of a size, η, and lifetime, t_η, given by the Kolmogorov length and time scales, respectively.

It is postulated that burning within both types of eddy is essentially a molecular phenomenon, as assumed by Kuznetsov (1976), and simplification of the chemical kinetics is achieved by the use of laminar burning velocity as an expression of chemical reaction rate. As in other eddy decay theories, because mixing at the molecular level is ultimately the agency by which the flame front is propagated, the rate of burning is expressed in terms of the rate of decay of eddies. However, in the present approach, this involves two types of eddy and the coupling of eddy decay rate with the amount of chemical reaction during an eddy lifetime. The rate of burning, for each eddy size, is expressed by the product of the rate of eddy decay and the amount of mixture chemically reacted during the eddy lifetime.

Chemical lifetimes for the complete combustion of eddies must be assigned to the two types of eddy. Experiments on turbulent flames (Abdel-Gayed *et al.* 1979) support the suggestions of Tennekes (1968) and Kuo & Corrsin (1972) for isothermal turbulence: that dissipative eddies consist of vortex tubes, with a diameter of the order of η, and a spacing of the order of the Taylor microscale, λ. This structure is discussed further in § 3(*d*).

It might be expected that laminar flame propagation across a large eddy would originate at the dissipative vortex tubes. If the average distance between these is λ, then the distance the flame must travel to react the mixture is 0.5 λ and the chemical lifetime becomes 0.5 λ/u_l. This seems more realistic than a lifetime of L/u_l, which has been suggested previously (Abdel-Gayed *et al.* 1979), and leads to burning velocities in better agreement with the experimental data. It is difficult to assign a chemical lifetime to dissipative eddies and there are few experimental data to aid the resolution of this problem. If laminar burning across vortex tubes is assumed, the average distance of flame propagation is 0.5 η and the chemical lifetime is 0.5 η/u_l.

As is the case with other theories of turbulent burning, there are many uncertainties, but currently there is some justification in postulating physical models that might ultimately be clarified by further experimental studies.

An experimental p.d.f. is assumed for eddy lifetimes (Roshko 1976), but a constant value is assumed for chemical lifetimes. It is convenient to base all the rate processes upon a common dimensionless time, $\tau = u'^2t/\nu$, where t is the real time. Turbulence is assumed to be uniform and isotropic.

(*c*) *Eddy burning rates*

The dimensionless mean lifetime of large eddies, τ_{dL}, becomes

$$\tau_{dL} = u'^2L/\nu u' = R_L. \tag{8}$$

A decay constant, κ_L, which is the reciprocal of τ_{dL} may be defined. Thus

$$\kappa_L = R_L^{-1}. \tag{9}$$

If there are n_L large eddies in unit volume, their volumetric decay rate is given by $\kappa_L n_L$.

Let p_L be the probable fraction of an eddy that burns during the eddy lifetime and V_L be the eddy volume. If r is the mean reactedness at a point in the flame, then $1-r$ is the mean fractional volume unreacted. More rigorously, r should be expressed by a probability density function, but here a time mean value will suffice. The associated dimensionless volumetric rate of burning, B_L, becomes

$$B_L = \kappa_L n_L p_L V_L(1-r). \tag{10}$$

This form of expression is adopted for both large and small eddies. This is because, for chemical reaction to be effective in flame propagation, it must be followed by the break-up of the eddy in which it occurs, and ultimately by the mixing of the molecules that comprise it with those from other eddies.

The value of p_L depends upon the dimensionless chemical lifetime, τ_{cL}, for a large eddy and this is given by

$$\tau_{cL} = \frac{c_L \lambda}{u_l} \frac{u'^2}{\nu}, \tag{11}$$

where c_L is a numerical constant, taken to be 0.5 throughout the present work, which depends upon the geometry of large eddy burning.

For conditions appropriate to the present analysis, the Taylor microscale, λ, can be related to the integral length scale, L, by

$$\lambda^2/L = A\nu/u' \tag{12}$$

in which A is a numerical constant. Experimental values of L/λ have been plotted against a turbulent Reynolds number based on λ (Abdel-Gayed & Bradley 1977 *b*), together with additional points from grid-generated turbulence (Comte-Bellot & Corrsin 1971) and values taken from Rotta (1951). For values of $R_L > 60$ these gave a value of A equal to 40.4, and this is used throughout the present work.

With this value, equations (11) and (12) give

$$\tau_{cL} = c_L 6.36 R_L^{0.5} u'/u_l. \tag{13}$$

The fraction of an eddy that has reacted during an eddy lifetime of τ ($\leqslant \tau_{cL}$) will, in general, be linearly related to time and be given by τ/τ_{cL}. For $\tau > \tau_{cL}$ all the eddy will have reacted. With a p.d.f. for eddy lifetimes given by $\tau_{dL}^{-1} \exp(-\tau/\tau_{dL})$,

$$p_L = \int_0^{\tau_{cL}} \frac{\tau}{\tau_{cL}} \tau_{dL}^{-1} \exp(-\tau/\tau_{dL})\, d\tau + \int_{\tau_{cL}}^{\infty} \tau_{dL}^{-1} \exp(-\tau/\tau_{dL})\, d\tau, \tag{14}$$

$$p_L = [1 - \exp(-\tau_{cL}/\tau_{dL})]\, \tau_{dL}/\tau_{cL}. \tag{15}$$

It is assumed that n_L is related to the intermittency factor for large eddies, γ_L, by

$$n_L = \gamma_L/V_L. \tag{16}$$

Equations (8)–(16) yield

$$B_L = [1 - \exp\{-c_L 6.36 R_L^{-0.5}(u'/u_l)\}]\, \frac{1-r}{c_L 6.36 R_L^{0.5}(u'/u_l)}. \tag{17}$$

For small eddies the Kolmogorov time scale is given by (Andrews *et al.* 1975 *b*)

$$t_\eta = \lambda/15^{0.5} u'. \tag{18}$$

From equations (12)–(18) the dimensionless mean lifetime of small eddies, $\tau_{d\eta}$, becomes

$$\tau_{d\eta} = (u'^2/\nu)\, t_\eta = 1.641 R_L^{0.5}. \tag{19}$$

The associated decay constant, κ_η, is the reciprocal of $\tau_{d\eta}$, and

$$\kappa_\eta = 0.609 R_L^{-0.5}. \tag{20}$$

The dimensionless chemical lifetime for a dissipative eddy is given by

$$\tau_{c\eta} = c_\eta \eta u'^2/u_l \nu, \tag{21}$$

where c_η is a numerical constant, taken to be 0.5 throughout the present work, which depends upon the geometry of small-eddy burning. The Kolmogorov length scale, η, is given by (Andrews *et al.* 1975 *a*)

$$\eta = (A/15)^{0.25} L R_L^{-0.75}. \tag{22}$$

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With $A = 40.4$, equations (21) and (22) yield

$$\tau_{c\eta} = c_\eta 1.281 R_L^{0.25} u'/u_l. \tag{23}$$

The dimensionless volumetric rate of burning in these eddies, B_η, is given by

$$B_\eta = \kappa_\eta n_\eta p_\eta V_\eta (1-r) \tag{24}$$

in which n_η is the number of small eddies, in unit volume, p_η is the probable fraction of an eddy which burns during the eddy lifetime and V_η is the eddy volume.

The evaluation of p_η follows along similar lines to those used for p_L. It is assumed again that the fraction burnt is linear with time up to the chemical lifetime, $\tau_{c\eta}$. The p.d.f. for eddy lifetime is given by $\tau_{d\eta}^{-1} \exp(-\tau/\tau_{d\eta})$ and by following equations (14) and (15)

$$p_\eta = [1-\exp(-\tau_{c\eta}/\tau_{d\eta})]\, \tau_{d\eta}/\tau_{c\eta} \tag{25}$$

and

$$n_\eta = \gamma_\eta / V_\eta, \tag{26}$$

where γ_η is the intermittency factor for small eddies. Equations (19)–(26) yield

$$B_\eta = \gamma_\eta [1-\exp\{-c_\eta 0.781 R_L^{-0.25}(u'/u_l)\}] \frac{1-r}{c_\eta 1.282 R_L^{0.25}(u'/u_l)}. \tag{27}$$

(*d*) *The intermittency of small eddies*

A knowledge of the intermittency factors is necessary for the evaluation of both B_L and B_η. With regard to the former, for the conditions of fully developed turbulence considered here, the intermittency factor, γ_L, of the large eddies may be taken as unity. This, of course, does not preclude the existence of small dissipative regions within an overall larger eddy structure.

The evaluation of the factor for the small eddies presents greater problems. The experimental observation of intermittency led Townsend (1951) to postulate a turbulence structure in which small-scale eddies are represented as a random tangle of vortex sheets (locally parallel vortex lines). Corrsin (1962) suggested that the vortex sheets have a thickness of the order of the Kolmorogov microscale, η, and a spacing of the order of the integral scale, L, and that these dissipative regions occupy a volume fraction of order η/L. From equation (22), with A equal to 40.4, it is readily shown that the fractional volume occupied by small eddies is given by

$$\gamma_\eta = 1.281 R_L^{-0.75}. \tag{28}$$

Saffman (1968), in an approximate heuristic analysis, assumed the turbulent energy to be dissipated in concentrated sheets and tubes of vorticity. According to him either the thickness of the sheets, or the radius of the tubes, has a characteristic length scale, δ, and this is less than the Taylor microscale. For the sheets, the proportion of the volume occupied by them is shown to be $(\delta/L)^{\frac{1}{2}}$. For the tubes, this proportion becomes δ/L. He postulated that the somewhat organized sheets and tubes are themselves unstable and degenerate into small scale motions. For example, it is known that curved vortex sheets may develop Taylor–Görtler instabilities and that tubes may develop Taylor–Couette instabilities. In both cases a stable secondary motion ensues with a cellular structure of size approximately δ. Saffman showed boundary layers to exist at the cell edges, and the fractional volume of the sheets and tubes occupied by such regions of concentrated vorticity to be η/δ.

The fractional volume occupied by these strongly dissipative regions in the case of sheets will be equated to γ_η and is given by

$$\gamma_\eta = \left(\frac{\delta}{L}\right)^{0.5}\frac{\eta}{\delta} = \frac{\eta}{(L\delta)^{0.5}}. \tag{29}$$

Saffman shows δ to be given by

$$\delta = (\nu L/u')^{\frac{1}{2}}. \tag{30}$$

Equations (12) and (30) give

$$\delta = 0.157\lambda. \tag{31}$$

Equations (12), (22), (29) and (31) yield

$$\gamma_\eta = 1.281 R_L^{-0.5}. \tag{32}$$

In the case of tubes, the fractional volume occupied by the dissipative regions is (δ/L) (η/δ) or η/L. This leads to the same expression, equation (28), as that of Corrsin for dissipative sheets.

Tennekes (1968) has suggested that dissipative eddies consist of vortex tubes with a diameter of the order of the Kolmogorov microscale and a spacing of the order of the Taylor microscale, λ, and that these vortex tubes occupy a volume fraction η^2/λ^2. From equations (12) and (22) this leads to

$$\gamma_\eta = 40.62 \times 10^{-3} R_L^{-0.5}. \tag{33}$$

More recently, Kuo & Corrsin (1972) have attempted to identify experimentally the geometric character of the regions of random fine structure. They tentatively have concluded that these regions are more likely to be rod-like than blob-like or slab-like and suggest that random, slightly 'stringy' structures might overlap each other. The average linear dimensions of these regions of fine structure are considerably larger than the turbulent fine structure within them.

In terms of the present two-eddy theory, a value of γ_η may be derived from the assumed equilibrium between the two groups of eddies. The large eddies break down to small eddies, the motion of which, in turn, is dissipated into molecular motion. If it is assumed that all the matter in the large eddies eventually comprises that of small eddies, upon the breakdown of the former, then the volumetric rate of formation of small eddies may be equated to their rate of dissipation, or

$$\kappa_L n_L V_L = \kappa_\eta n_\eta V_\eta. \tag{34}$$

From equations (16), (26) and (34),

$$\kappa_L/\kappa_\eta = \gamma_\eta/\gamma_L. \tag{35}$$

As previously discussed, γ_L may be taken to be unity and from equations (9), (20) and (35),

$$\gamma_\eta = 1.641 R_L^{-0.5}. \tag{36}$$

Thus all the theoretical predictions of the fractional volume occupied by the small strongly dissipative regions and given by equations (28), (32), (33) and (36) show this proportion to decrease as R_L increases. With the exception of the first, all these equations show an inverse square root dependency of the proportion upon R_L. Calculated values of small-eddy fractional volume for two typical values of R_L are given in table 2. This shows that the fractional volume might be less than 1%, and that there can be a thirty-fold variation in the value given by the different expressions.

(e) *Turbulent burning velocities*

The two-eddy theory of burning can yield an approximate expression for the turbulent burning velocity. In a Lagrangian sense, the total rate of burning $(B_L + B_\eta)$, can be expressed as the rate of change of reactedness, r. For a one-dimensional flame, diffusion being neglected,

$$\mathrm{D}r/\mathrm{D}\tau = B_L + B_\eta \tag{37}$$

or

$$\mathrm{D}r/(1-r) = (B_L + B_\eta)\,\mathrm{D}\tau/(1-r). \tag{38}$$

In the present simplified analysis $(B_L + B_\eta)(1-r)^{-1}$ is not a function of reactedness and the equation is readily integrated. In a more rigorous analysis this would not be so, and p.d.fs would be assigned to both chemical lifetimes and reactedness. Additionally, instead of cold flow turbulence parameters, allowance might be made for the effects of combustion upon these.

Table 2. Fractional volume occupied by small dissipative eddies

author	equation no.	volume % occupied by small eddies $R_L = 500$	volume % occupied by small eddies $R_L = 5000$
Corrsin (1962) (sheets)	(28)	1.21	0.22
Saffman (1968) (tubes)	(28)	1.21	0.22
Saffman (1968) (sheets)	(32)	5.73	1.81
Tennekes (1968) (tubes)	(33)	0.18	0.06
present work	(36)	7.34	2.32

From equation (38), if τ_t is the dimensionless time for reactedness to progress from zero to a value of r_t, then

$$[(1-r)\ln(1-r_t)^{-1}]/(B_L + B_\eta) \tag{39}$$

and this can be evaluated from equations (17) and (27). Together with the two approximate expressions relating turbulent flame thickness, δ_t,

$$\delta_t \approx \epsilon/u_t \quad \text{and} \quad t_t \approx \delta_t/u_t, \tag{40), (41}$$

where ϵ is the turbulent diffusivity, it follows that

$$\frac{u_t}{u_l} \approx \frac{u'}{u_l}\left(\frac{\epsilon}{\nu\tau_t}\right)^{0.5} \tag{42}$$

and

$$\frac{\delta_t}{L} \approx \left(\frac{\epsilon\tau_t}{\nu}\right)^{0.5} R_L^{-1}. \tag{43}$$

The turbulent transport number, ϵ/ν, is related to the turbulent Reynolds number and, for isotropic turbulence, the authors (Abdel-Gayed & Bradley 1980) have proposed equation (3).

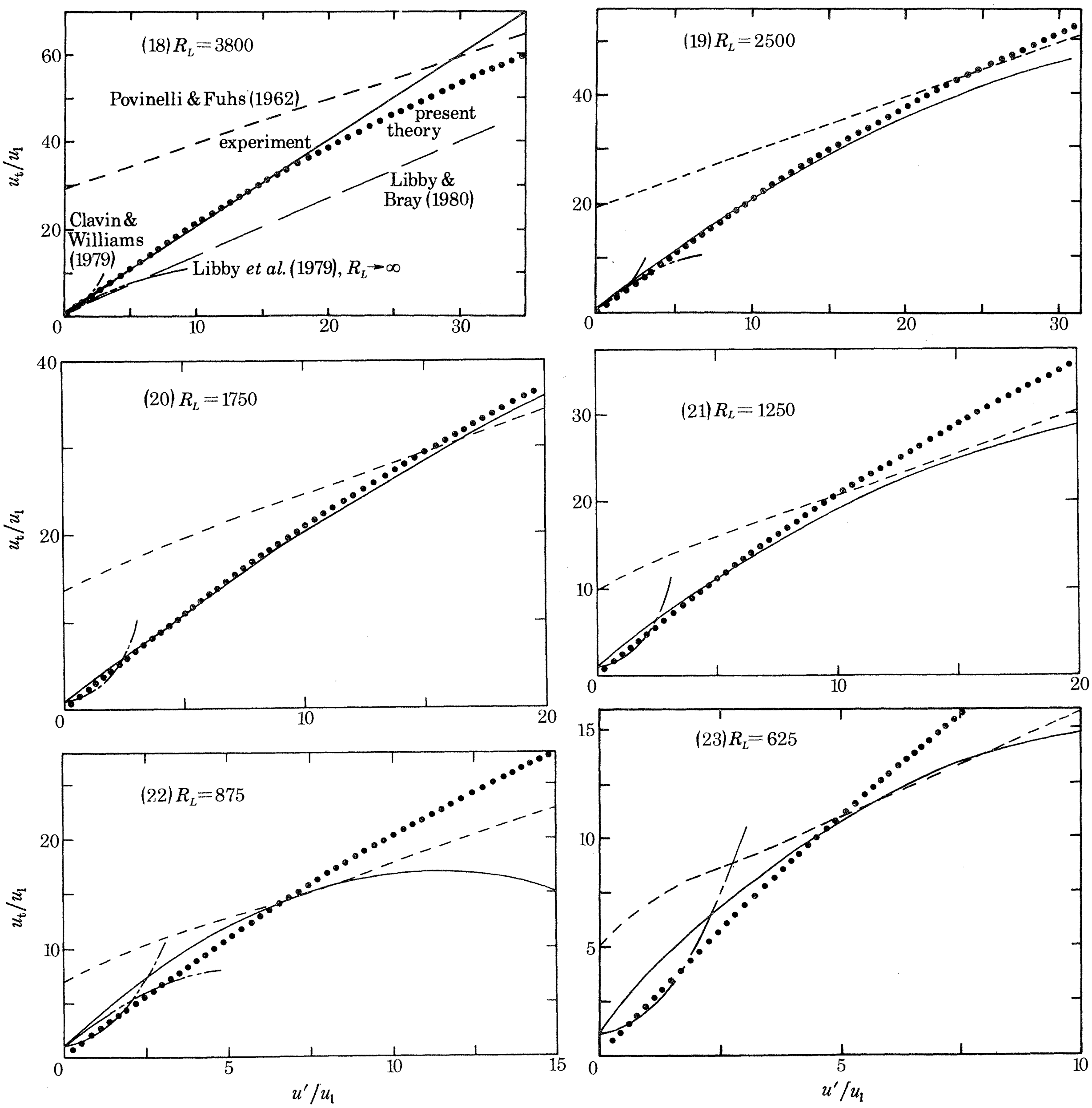
Values of u_t/u_l were found from equations (3), (17), (27), (39) and (42), with r_t equal to 0.90 and values of c_L and c_η equal to 0.5. The uncertainty of the theoretical values for intermittency are reflected in the different values given in table 2. It was assumed that γ_η varies inversely with the square root of the turbulent Reynolds number but that the constant could be different from that given by the different theoretical expressions. It was found that values of γ_η twenty times those given by equation (36) gave the best agreement between the theory and the experimental curves of figures 4–6. The same factor was used to increase B_L in equation (17), as an increase in the concentration of vortex tubes would increase the number of sources from which large eddy burning would start. Previous work suggested that, in general, the ratio B_η/B_L was greater than 10 (Abdel-Gayed *et al.* 1979). However, over a wide range of conditions the ratio was close to unity with the theory presented here.

With the above constants, the expressions for u_t/u_l and δ_t/L, which were used for the values given in figures 18–36, are:

$$\frac{u_t}{u_l} = \left\{1.34 R_L^{0.47}\frac{u'}{u_l}\left[1-\exp\left(-3.178 R_L^{-0.5}\frac{u'}{u_l}\right)\right] + 10.9 R_L^{0.22}\frac{u'}{u_l}\left[1-\exp\left(-0.391 R_L^{-0.25}\frac{u'}{u_l}\right)\right]\right\}^{0.5} \tag{44}$$

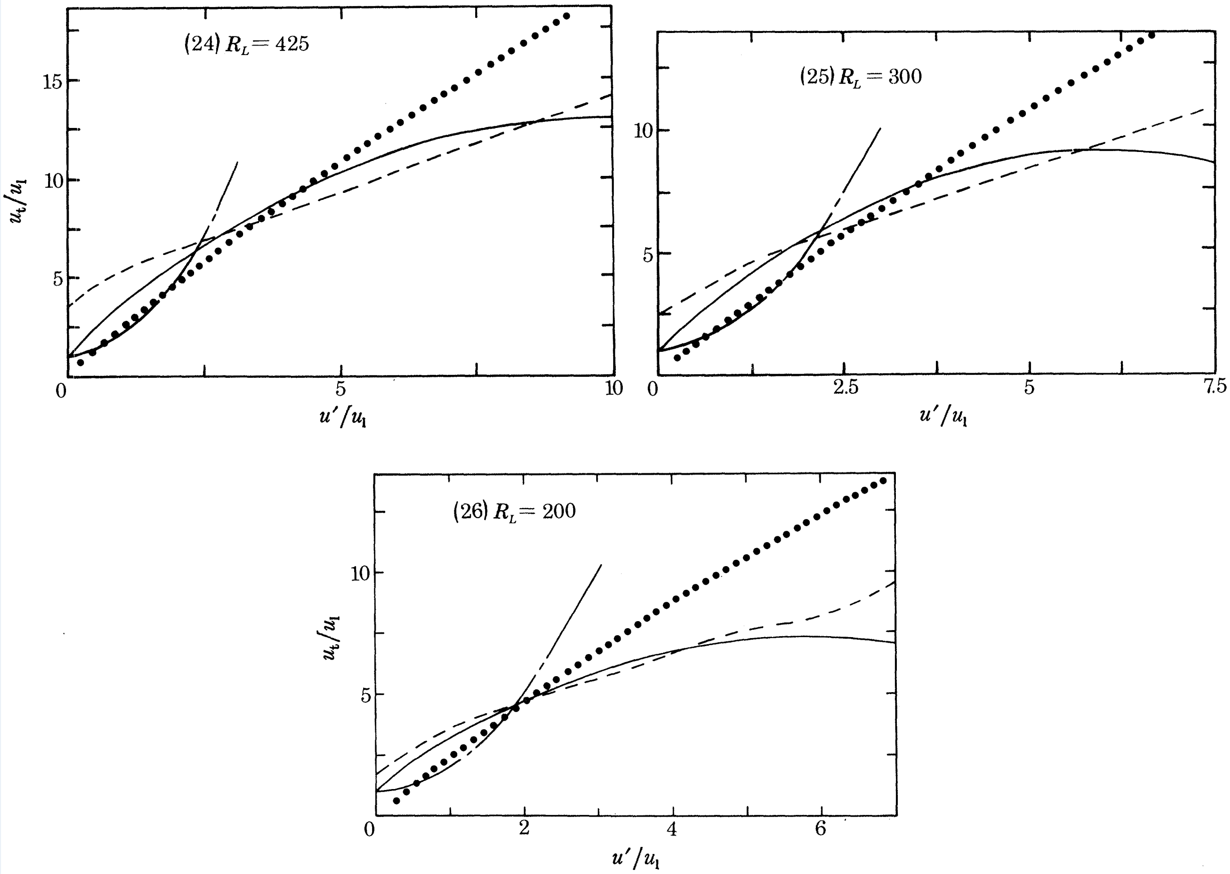
and

$$\frac{\delta_t}{L} = \left\{5.578\frac{R_L^{0.53}}{u'/u_l}\left[1-\exp\left(-3.178 R_L^{-0.5}\frac{u'}{u_l}\right)\right] + 45.38\frac{R_L^{0.28}}{u'/u_l}\left[1-\exp\left(-0.391 R_L^{-0.25}\frac{u'}{u_l}\right)\right]\right\}^{0.5}. \tag{45}$$

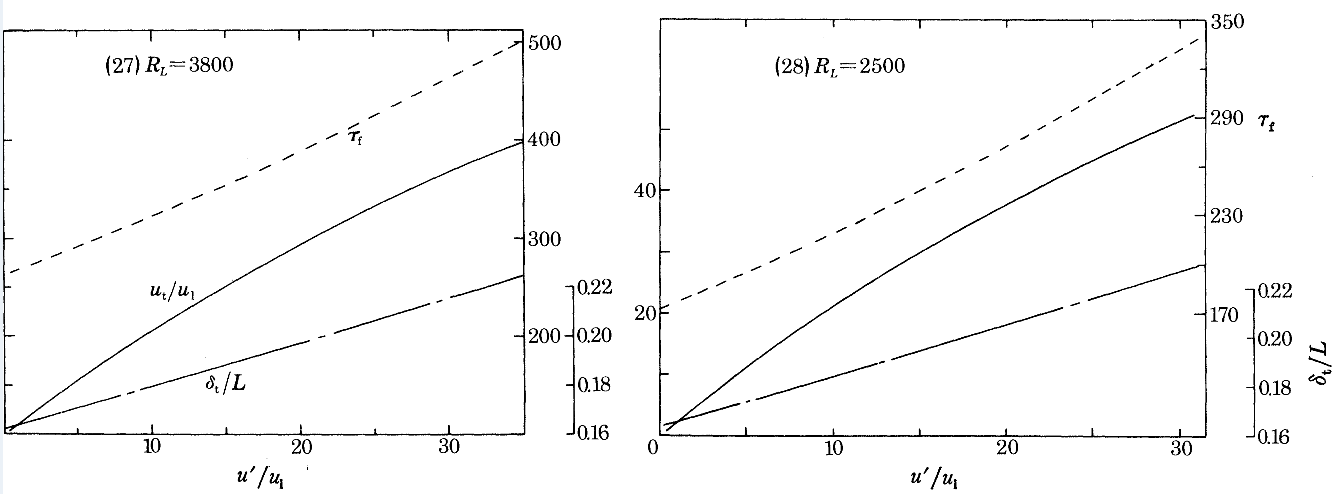


FIGURES 18–23. Comparison of experimental and theoretical values of u_t/u_l; key as in figure 18.

Figures 18–26 compare such theoretical values of u_t/u_1, shown by the dotted curves, with those of experiments, shown by the full line curves drawn from figures 4–12. Figures 27–35 show for nine different values of R_L not only the theoretical variations of u_t/u_1, but also those of τ_f and δ_t/L. Figure 36 summarizes the theoretical values of u_t/u_1 obtained from the two-eddy theory of burning.

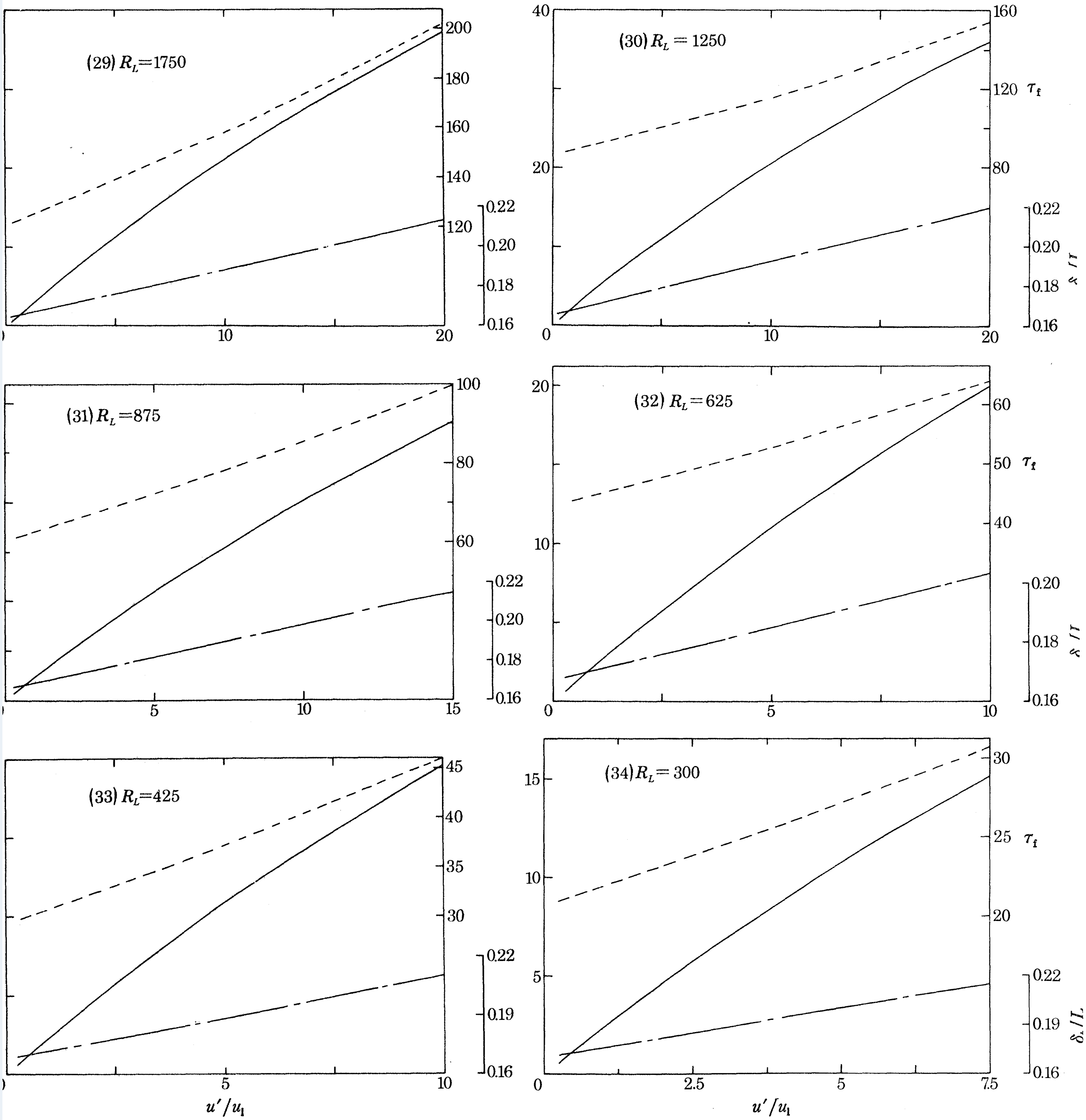


FIGURES 24–26. Comparison of experimental and theoretical values of u_t/u_1; key as in figure 18.



FIGURES 27 AND 28. Theoretical variations of u_t/u_1, τ_f and δ_t/L with u'/u_1; key as in figure 27.

The turbulent diffusivity increases with R_L, in accordance with equation (3), and this is a factor increasing the turbulent burning velocity. On the other hand, the dimensionless eddy decay constants, κ_L, and κ_η, decrease with R_L, in accordance with equations (9) and (20), and the small-eddy intermittency, γ_η, also decreases with R_L, in accordance with equation (36). These are factors decreasing the u_t/u_l ratio, as reference to equations (9), (20), (24), (26), (36), (39) and



FIGURES 29–34. Theoretical variations of u_t/u_l, τ_f and δ_t/L with u'/u_l; key as in figure 27.

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(42) makes clear. Thus, at the higher turbulent Reynolds numbers, the rate-limiting decay of eddies and the falling concentration of small eddies might be sufficient to decrease the propagation rate.

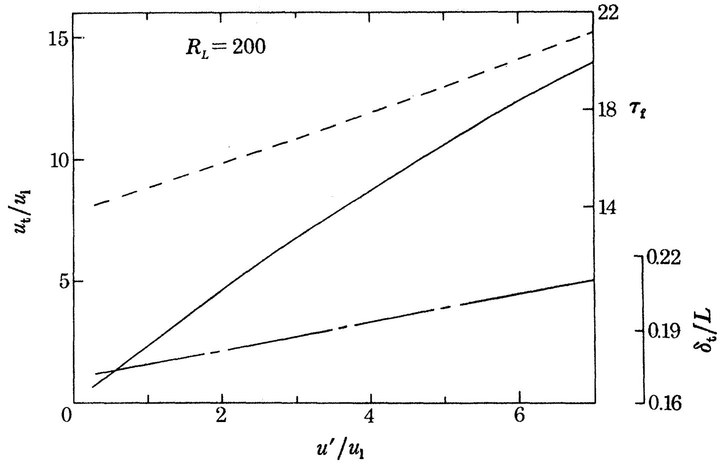

FIGURE 35. Theoretical variations of u_t/u_l, τ_t and δ_t/L with u'/u_l; key as in figure 27.

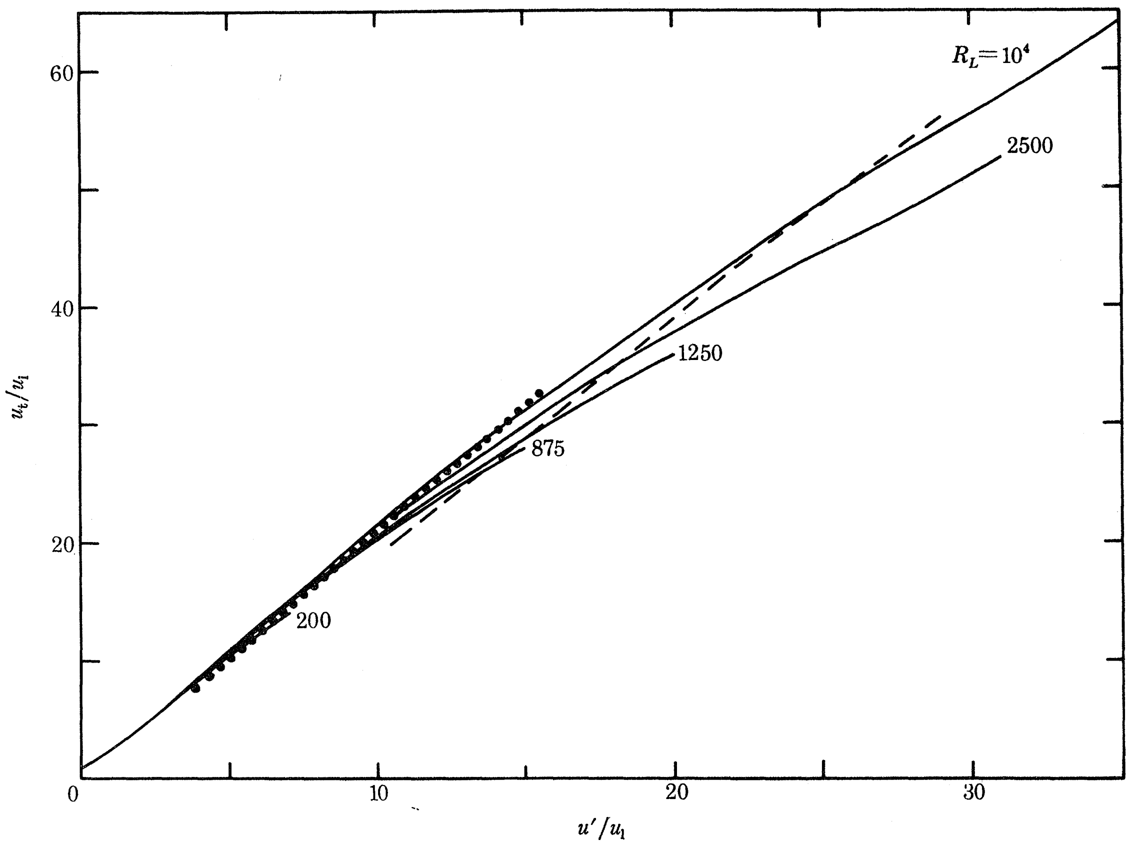

FIGURE 36. Variation of theoretical values of u_t/u_l with u'/u_l, for different values of R_L. Régimes of combustion are indicated (see §4.).

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4. Theoretical values of u_t/u_l compared

Since the work of Damköhler (1940) appeared, the ratio of the integral scale of turbulence to the laminar flame thickness and other similar ratios have been used as a criterion of the mode of flame propagation. A wrinkled flame condition is $L > \delta_l$ and an approximate expression for laminar flame thickness is (Gaydon & Wolfhard 1970)

$$\delta_l = \nu/u_l. \tag{46}$$

Hence, the wrinkled flame criterion becomes

$$u'/u_l < R_L. \tag{47}$$

This condition is fulfilled by almost all turbulent flames in practice.

Another approach to criteria for laminar flame wrinkling is that the reciprocal of the straining rate of the gas, which is given by the quotient of u' and a length scale, should be less than the residence time in the laminar flame, δ_l/u_l. Different turbulent length scales have been proposed to define the strain rate characteristic time. Kovasznay (1956) has suggested λ as an appropriate scale. Equations (12) and (46) then lead to the wrinkled flame condition

$$u'/u_l < 2.521 R_L^{0.25}. \tag{48}$$

Equality of the two sides of the equation is indicated by the dashed curve in figure 36. For values u'/u_l greater than the right-hand side, the rate of deformation can break up the flame front.

In another approach Klimov (1963, 1967) has taken L as the appropriate length scale which governs the straining rate and this leads to the wrinkled flame condition

$$u'/u_l < R_L^{0.5}. \tag{49}$$

Equality of the two terms occurs in a régime where values of u'/u_l are higher than those given in figure 36.

Williams (1976) has suggested as a criterion for a wrinkled flame structure that the laminar flame thickness must be smaller than η. For this condition equations (22) and (46) lead to

$$u'/u_l < 1.281 R_L^{0.25}. \tag{50}$$

Equality of these two terms is indicated by the dotted curve in figure 36.

Thus a variety of criteria have been proposed for the break-up of the flame front. Experimental studies, as evinced by the ciné-schlieren photographs of explosions in a stirred bomb by the authors of this paper, confirm that as the régime indicated by the dotted curve is approached, so the small scale eddy structure becomes more pronounced. There is, however, at the moment, little theoretical or experimental evidence of a sharp demarcation between régimes of wrinkled laminar flames and fragmented flames with a pronounced small eddy structure.

The theoretical approaches of other workers to the rate of premixed turbulent flame propagation are now reviewed briefly. The associated theoretical values of u_t/u_l are shown in figures 18–26, alongside those of the two-eddy theory and of experiment.

In their spectral theory of flame propagation Povinelli & Fuhs (1962) developed an analytical expression for u_t/u_l based on the sum of large- and small-scale effects. From measured turbulent energy spectra they evaluated two spectral weighting functions as 0.98 and 0.02 for the large- and small-scale influences, respectively. This leads to

$$\frac{u_t}{u_l} = 0.98\left(1+\frac{u'}{u_l}\right)+0.02\left(1+\frac{\epsilon}{\nu}\right). \tag{51}$$

In figures 18–26 this expression has been evaluated with values of ϵ/ν drawn from equation (3).

The results presented by Libby *et al.* (1979) for unconfined flames appear in figures 18, 19 and 22. Further results of Libby & Bray (1980), which show u_t/u_l to be independent of R_L, are given in figure 18.

For isotropic turbulence and small values of u'/u_l, the analysis of Clavin & Williams (1979) yields

$$u_t/u_l = 1+(u'/u_l)^2. \tag{52}$$

This relation is plotted in figures 18–26. The independence of u_t/u_l of R_L or, in dimensional terms, of any length-scale influence, in the régime of validity, is confirmed by the experimental results. The form of presentation previously adopted by the authors (Abdel-Gayed & Bradley 1977*a*), in which u_t/u_l was plotted again u_l/u', demonstrates this independence rather better than do the figures in the present paper. This is because of the elongation of the scale in the wrinkled flame region in the earlier paper. Although this work is informative for low levels of turbulence, it is not applicable at the higher levels.

At the lower values of R_L, reference to figures 18–26 shows that the values of u_t/u_l obtained from equations (3) and (51) give the best agreement with experiment, but at the higher values of R_L the results computed from the two-eddy theory give better agreement, provided u'/u_l is greater than unity. Differences between experimental and theoretical values reflect both the difficulties of accurate measurement and the formidable problems of theoretical formulation.

At present, all theories of turbulent burning are inherently limited by an incomplete understanding of the physical/chemical nature of the phenomenon and this is manifested mathematically in closure problems. Uncertainties arise in the two-eddy theory about the chemical lifetime in a dissipative eddy and the fractional volume occupied by such eddies. In the more mathematical analyses corresponding uncertainties arise about the chemical rate expression and probability density function. Despite these difficulties, three dimensionless parameters have emerged for the description of turbulent premixed combustion and the principal physico-chemical phenomena involved in it.

5. Conclusions

(i) New correlations of turbulent transport and flow parameters have been used to improve the correlation of experimental values of turbulent velocity. Some new data are presented in a form that supports the correlation of u_t/u_l with u'/u_l and R_L. Most of the known burner measurements of u_t are presented and it is shown that they can be correlated in this way.

(ii) The two-eddy theory of burning has been developed further. Uncertainties remain with regard to the small-scale turbulent structure and the mode of combustion in it.

(iii) Different theories for turbulent burning velocities have been examined and their predictions compared with practice. The two-eddy theory seems satisfactory for the higher levels of turbulence.

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